Physics III
ISI B.Math
Final Exam : November 12 ,2018

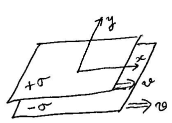
Total Marks: 75
Time : 3 hours
Answer all questions

1. (**Marks: 7 + 8 = 15**)

(i) A ring of radius R has a total charge $+Q$ uniformly distributed on it. Calculate the electric field and the potential at the centre of the ring.

(ii) Consider a negative charge $-Q$ constrained to slide along the axis of the ring. Show that the charge will execute simple harmonic motion for small displacements perpendicular to the plane of the ring.

2. (**Marks: 4 + 4 + 3 + 4 = 15**)



(i)An infinite uniform surface current $\mathbf{K} = K\hat{\mathbf{x}}$ flowing over an infinite sheet in the $x - y$ plane. Show that its magnetic field will be in the **y** direction and find its magnitude using Ampere's law with an appropriate Amperean loop.

(ii) A large (effectively infinite) parallel plate capacitor with uniform surface charge σ on the upper plate and $-\sigma$ on the lower plate is moving to the right with a constant speed v as shown in the figure.

(a) Find the magnetic field between the plates and also above and below them.

(b) Find the magnetic force per unit area on the upper plate, including its direction.

(c) At what speed v would the magnetic force balance the electrical force ?

[Hint: Use the result in part (i) to work out part (ii)]

3. (**Marks: 7 + 8 = 15**)

(a) A square loop of wire of side a, lies midway between two long wires, $3a$ apart and in the same plane.(Actually, the long wires are sides of a large rectangular loop, but the short ends are so far away that they can be neglected). A clockwise current I in the square loop is gradually increasing : $\frac{dI}{dt} = k$ where k is a constant. Find the emf induced in the big loop and the mutual inductance. Which way will the induced current flow ?

(b) Show that the magnetic vector potential $\mathbf{A}$ defined through $\mathbf{B} = \nabla \times \mathbf{A}$ is not unique and show that this non-uniqueness can be exploited to set $\nabla \cdot \mathbf{A} = 0$. For a uniform magnetic field it is given that $\mathbf{A}(\mathbf{r}) = -\frac{\mathbf{1}}{\mathbf{2}}(\mathbf{r} \times \mathbf{B})$.Show that it is indeed true that $\nabla \cdot \mathbf{A} = 0$

4. (**Marks: 4 + 3 + 4 + 4 = 15**)

(a) Write down the full set of Maxwell's equations for the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ in the presence of a charge density ρ and current density $\mathbf{J}$.

(b) Derive the continuity equation relating the charge density ρ and current density $\mathbf{J}$ from Maxwell's equations. Explain how the continuity equation is an expression of the local conservation of charge.

(c) Show that a magnetic field $\mathbf{B}$ and electric field $\mathbf{E}$ that is a solution to Maxwell's equations can always be written as

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}$$

where ϕ is a scalar function and $\mathbf{A}$ is a vector field.

(d) Starting from source free Maxwell's equations in vacuum, show that each component of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ obeys a wave equation. Find the velocity of the wave in terms of ϵ_0 and μ_0.(Note that :$\nabla \times \nabla \times \mathbf{A} = -\nabla^2 \mathbf{A} + \nabla(\nabla \cdot \mathbf{A})$)

5. (**Marks: 3 + 3 + 4 + 3 + 2 = 15**)

(a) Write down a monochromatic plane wave solution to the vacuum Maxwell's equations for the electric field with an amplitude E_0, wave number k, angular frequency ω , zero initial phase angle that is propagating in the x direction and polarized along the direction of the unit vector $\hat{\mathbf{n}}$.

(b) Using Maxwell's equations, show that $\hat{\mathbf{n}}$ must lie in the $y - z$ plane.

(c) If B_0 is the amplitude of the corresponding plane wave solution for the magnetic field with the same frequency propagating in the same direction, show that $B_0 = \frac{E_0}{c}$. If $\hat{\mathbf{n}}$ lies along the y axis find the direction of $\mathbf{B}$

(d) Find the average energy density of the above electromagnetic wave (include the electric and magnetic contributions) in terms of E_0.

(e) In which direction will the Poynting vector of the above electromagnetic wave point ?